The Divine Nexus of Music and Mathematics

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I have this fantasy.

I'd love to see a debate on a liberal arts college campus between members of the sociology department, those who bemoan the heritage of European culture at every turn, and members of the music faculty who revere the music of Bach, Chopin and Wagner.

It would be fascinating to witness the spectacle of the sociology contingent trying to convince the musicians that they have it all wrong regarding the music of their cherished composers.

But do musicians in the academy really have it all wrong regarding Western culture? Are those who argue there may be "immutable truths" that govern music — its creation and realization — completely obtuse to sociological or cultural prejudices as postmodernists would have us believe?

Though we might debate the cosmological and metaphysical aspects of music, a cursory examination of music (regardless of cultural sphere) reveals that the laws and principles that govern music production are rooted in mathematics and physics.

This understanding dates back to Pythagoras in Greece and the Sumerians of Mesopotamia. Nevertheless, we have come to a point in our postmodern culture where any allusion to "certainty," "universals" or "immutable truths" is, more often than not, met with skepticism, even abject derision.

This mindset originated with Nietzsche, Marx, Stirner, Hegel, and others whose abnegation of religion and "absolutes" infected Western culture at the beginning of the 20th century.

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That pernicious legacy persists today.

Paradoxically, those who disparage religion love the music that is the progeny of a decidedly religious culture. Nietzsche, a cultural revolutionary and an earlier admirer of Richard Wagner's operas, said, "Life without music would be a mistake."

In *Dionysos Rising: The Birth of Cultural Revolution Out of the Spirit of Music* (1994), Catholic scholar E. Michael Jones alludes to the views of music by the cultures of antiquity:

According to both the ancients and their Christian followers, the order of the creation was love, bound together in a unity both mathematical and musical. Indeed, love, divine order, music and mathematics are simply the four different ways of saying the same thing.

Historically, philosophical debates regarding the governing principles of heaven and earth have played heavily into our understandings of art and music. Though we cannot know the exact origins of music, we do know it has always been realized according to mathematical properties — frequencies, ratios, intervals, durations and decibels. Sound vibrations can be explicated mathematically according to physical law via the overtone series, and the combinatory utilization of these properties, whether codified or not, point to mathematics as the basis of sound and music on the corporeal level.

Our utilization of these various properties in imaginative ways leads us to the creation of music. An isolated pitch-event or vibration may not evoke an emotional response, but the combining of pitches in an orderly and principled fashion surely can. This begins to explain how something as abstract and ephemeral as music can be simultaneously aesthetically pleasing (subjective) and logical (objective), thereby possessing the ability to speak to our heart and mind — the totality of our personhood.

As Unificationist theologian, Dr. Young Oon Kim states in *Unification Theology and Christian Thought*, it is the principled application and utilization of musical materials that puts us in touch with divine essence — God's and our own.

It is in the transmoral dimension of aesthetic experience that beauty approaches God. All the laws from

and within God — give and take, polarity, harmony – connect beauty from all cultures. And to the extent that they clearly amplify and substantiate God's nature, they evoke a response of love and appreciation from man. Since God represents absolute love and freedom, beauty is never confined.

Numerology

The study of numerology has been associated with the occult, astrology, astronomy, the paranormal, the divinatory arts and "New Age" spirituality. However, the scriptural teachings of Judeo-Christian theology and Divine Principle are inextricably linked to numerology as well: The Trinity, Jesus' 40-day fast, Jacob's 21 years in Haran, the 40-day flood judgment, God resting on the seventh day, the eight Beatitudes, Jesus' 12 apostles and 72 disciples, the 12 gates of heaven, Jesus' three temptations, the 12 tribes of Israel, the three archangels, the three wise men, Peter's three denials, etc.

It's intriguing to note that these scriptural numbers are various multiples of 3 and 4. Unificationist doctrine refers to the number 3 as being representative of heaven (the Trinity, Jesus resurrected on the third day, etc.) and the number 4 being representative of earth and nature (the four seasons, the four trade winds, the four directions.) Jewish prayers are often repeated in sequences of three and seven. Pythagoras believed that the number 3 was more significant than the number 2 due to the implication of a beginning, middle and end — three stages of development. The "prime number" 5 (2+3) also has distinct musical implications. Muslims offer prayer (*Salāh*) five times a day.

The acoustic principles of pitch production are defined by mathematical ratios, and as the philosophers and theorists of antiquity would eventually realize, mathematics was intrinsic to virtually every aspect of the natural world, including music**.** Pythagoras' epiphany about numbers being "the stuff of the universe" was proven to be correct and opened the doors for physicists and theorists to understand the "secrets" of the created world with greater clarity and vision.

The ramifications of his findings as it pertained to the art of music would have major implications, as Pythagorean theory would be the basis upon which musicians carried out their creative endeavors for centuries. Though some consider numerology to be pseudo-mathematics, nature tells us otherwise. Albert Einstein (who played violin and piano) posited that the laws of gravity are predicated on the idea that there exists a geometric link between time and space. The nexus is real.

David Eaton performs with actor/singer Robert Davi and the New York City Symphony at the United Nations' 70th Anniversary concert on June 30, 2015.

Numbers Don't Lie

Historically, there have been a numerous attempts to organize musical materials according to the laws of the natural world. Leonard Bernstein referred to the pentatonic scale as being "humanity's favorite scale." This five-note scale, "handed to us by nature," is based on the first five *differing* pitches in a given overtone series and is common to the music of Asia, Ireland, Eastern Europe and Africa.

Diatonic tonality (predicated on a seven-note scale) evolved from the monophonic music of the early Christian church in Europe, and although this type of tonal centricity can also be found to varying extents in the folk music of Asia, Africa and the Middle East (*maqam*, the seven-note Arabic scale), it came to its full flowering in Europe. As musical theory developed in the West, the numerological properties governing the musical organization of sound and rhythm serendipitously reflected the numerology found in scripture. Was this purely coincidental, or was there an implicit design in the equation?

In musical parlance "tonality" refers to the codified system of pitch and chord relationships that results in a specific hierarchical syntax that induces *aurally perceived* stabilities and attractions. As composers in Europe during the Renaissance began to write music having greater linear complexity (polyphony), the natural by-product of this process was a vertical alignment of tones that possessed definite harmonic modalities. The evolution of harmonic syntax though the 15th and 16th centuries, along with the experiments in tuning and interval modification (temperament) led to the development of specific tonal theories which gave rise to an ordered system of major/minor key centers — a manifestation of yang and yin — polar opposites. As it was in the evolution of natural languages, the "rules" that govern musical grammar grew out of the subconscious and were abstracted after usage, not before.

How does all this relate to numerology? Consider the following numerological and relational properties in Western tonality.

In the tonal syntax there are two basic modalities: major and minor. The octave in Western music is divided into 12 equal parts (semitones). There are seven pitches in the diatonic scale, with the eighth pitch being the start of a new octave — the proverbial "new beginning." There are seven "flat" major keys and seven "sharp" major keys and each major key corresponds to a relative minor key. As previously mentioned, the basis of Western harmony is the triad (the vertical organization of three pitches). The number 7 is also evident in the overtone series (the physical principles that govern tone production) as the first two octaves based on any "fundamental" tone consists of seven pitches (known as partials). Interestingly, the sixth partial in the overtone series is irregular in that it is mathematically imprecise in its relation to its "fundamental" tone.

Musical architecture and form also embody numerical aspects. *Sonata form*, a musical structure that evolved in the 18th and 19th centuries, is a musical form that encompasses a dual-key tonal framework and has three distinct sections: the exposition, the development, and the recapitulation. This tripartite musical structure reflects Pythagoras' views regarding the number 3 and is in accord with the three stages of growth as defined in Divine Principle.

In the musical settings of the Catholic mass in the 18th and 19th century, most notably those by Mozart and Haydn, the *Gloria* (which praises the Almighty) was often, though not exclusively, composed using 3/4 time, whereas the *Credo* (which was man's statement of faith), was composed in 4/4 time; heaven and earth being represented by the numbers 3 and 4. Moreover, in the tonal spectrum, a given major triad can have four distinct functions depending on its hierarchical position in a specific key, thus manifesting the "triple object purpose" as defined in Divine Principle. In the modern symphony orchestra there are three groups of instruments of definite pitch (strings, woodwinds and brass) and four "voices" within each group that correspond to the four basic ranges of the human voice: soprano, alto, tenor and bass.

Examples of polarity and relatedness are highly evident in the harmonic grammar of Western tonal music, including the polarity of major and minor keys, major and minor triads within those keys, tonic and dominant harmonies, consonant and dissonant intervals, whole-tones and semi-tones, diatonic and chromatic. In addition to these harmonic properties there are other polar opposites that come into play when creating or performing music: fast and low tempos, long and short durations, high and low pitches, adjacent and non-adjacent tones, loud and soft volume (dynamics), getting louder (*crescendo*) and getting softer (*diminuendo*), tempo acceleration (*accelerando*) and tempo deceleration (*ritardando*), bright timbres and dark timbres, duple meters (2/4, 4/4, 2/2) and triple meters (3/4, 6/8, 9/8), symmetrical meters (2/4, 4/4, 2/2) and asymmetrical meters (5/4, 5/8, 7/8).

When these various polar characteristics are manifested in a harmonized fashion, music begins to take on attributes that reflect divinity and possess great beauty and meaning. St. Paul writes in Romans 1:20 that God's invisible nature and deity can be clearly perceived in the created world; hence music that embodies the characteristics found in nature can be said to have "godly" attributes, as Dr. Kim asserts. Though all musical systems are human constructs, it can be construed that all music is predicated on divine design via nature.

Arnold Toynbee, in his 12-volume *A Study of History*, famously averred that the Church was the chrysalis out of which Western culture emerged. Because Christianity flourished in Europe, we can extrapolate that God vouchsafed His blessing by way of revelations to various believers in the realms of physics, science, music, art, architecture, literature, poetry, medicine, and astronomy. The evolution of the syntax of tonality was a significant aspect of the flowering of that deep-seeded religious impulse.

Numerology helps tell the story.

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