## PARALLELS BETWEEN

## UNIFICATION THOUGHT AND NUMBERS



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In the 2007 edition of the Journal of Unification Studies I wrote an article with the title "The Yin and Yang of Prime Numbers: Finding Evidence of Unification Thought's Teachings on the Dual Characteristics in Prime Number Reciprocals". ${ }^{2}$ It provided a rather general introduction to my findings in the world of numbers.

Despite some unavoidable overlap with that article, the current focus is solely on the two types of dual characteristics as explained in Rev. Sun Myung Moon's Unification Thought and Divine Principle and how it parallels with my number research findings.

Friends of mine ask me occasionally what the practical applications of my numeric discoveries are, almost like telling me "So what?" While I understand these concerns, there is more to life than just pragmatism. We also benefit in life from human created beauty, such as art, music, poetry, dance, architecture, as well as from divinely created natural beauty like flowers and sunsets, and it is my firm conviction that when the inherent beauty and structure of numbers is revealed that a person's grasp of

[^0]math will improve, which is actually of great pragmatic benefit. Consider the fact that $70 \%$ of students continue to fail at math.
An article on number theory, a branch of mathematics, in a theological journal may seem out of place, and yet we will see that it isn't. After all, Rev. Moon himself has repeatedly emphasized the importance of numbers, especially during his final earthly years, as he saw them in a providential light. It would make for fascinating reading if someone were to thoroughly study Rev. Moon's reading on the significance of numbers.

On my side, I have been simply studying the very nature and characteristics of numbers themselves, from a purely arithmetic and algebra-based analytical and intellectual perspective. I therefore assign no particular meaning to any number, like 12,40 , etc. Generally, number theory is not to be confused with numerology.

## Two Types of Dual Characteristics

Anyone familiar with Unification Thought knows that Rev. Moon has taken the ancient Oriental philosophical notion of the dual characteristics of Yang and Yin ${ }^{3}$ quite a step further, by making a further delineation, as all Yang and Yin (e.g. a man and a woman) also each possess another pair of dual characteristics, namely that of internal character (mind/spirit) and external form (body/matter), which in Korean is called Sung Sang (invisible element) and Hyung Sang (visible element). What fascinated me was the discovery that these two-fold dual characteristics also apply to the world of numbers!

What follows below is an introductory encapsulation of these findings as they relate to this notion of two-fold dual characteristics, which are explored fully in my book The Secret World of Numbers. In Unification Thought the topics of Subject (initiating force) and Object (responding effect) between whom Giving and Receiving Action takes place, but these considerations are hard to discern in the world of numbers. I will try to touch upon these terminologies when I think it may be safe to do so.

At one point I was tempted to give my book The Secret World of Numbers the unusual title "The Psychology of Numbers", because, as I discovered, numbers behave in a certain fixed and even predictable way, and their behavior is also absolute. Rev. Moon, who repeatedly stressed the absoluteness of God and his Laws often asked scientists who gathered for the International Conferences on the Unity of the Sciences (ICUS) to center their papers on the theme of and quest for absolute values. Certainly, numbers belong absolutely to this realm. Also, for example, anyone studying the numbers ruling just our solar system will be intrigued by the numbers involved. ${ }^{4}$

[^1]
## PART ONE:

## Let's start at the very beginning

Like our abc's, we have to start with the numbers 0 through 9 , especially since all numbers, large and small, make use of these 10 digits $^{5}$. When we start to count, we start with single-digit numbers, but then we naturally soon expand to 2 -digits (e.g. 21, 28, 35, etc.) and many more digits when we start counting towards infinity. Thus the table of 7 looks look this: 7-14-21-28-35-42-49-56-63-70-77-84-etc.

Of course, this is elementary math. But to me, the important thing to observe is this: didn't we in fact start out with a single digit, namely that of 7 , that became two digits in 14 , 21 , etc.? We normally take this growth of the number of digits for granted, but looking at it from another perspective, we also know that the initial rule of the start of only one single digit has also been broken. Thus, I am taking the first number, a single digit, namely 7 , as my guide and starting point. But how can we turn all these double, triple, and many-fold digits into a single digit? Very simply through the process of adding the digits together until we can no longer do so, which is called the process of reduction. We reduct the digits to a final single digit. Thus 21 becomes $2+1=3$, and 35 becomes $8(3+5)$, etc. ${ }^{6}$

Thus, let's now move on to the table of 7 , from this new perspective, and it is going to look very different: $7(7=7), 5(14=1+4), 3(21=2+1), 1(28=2+8 ;$ then $1+0) ; 8(35=3+5), 6(42=4+2), 4(49=4$ +9 ; then $1+3), 2(56=5+6$, then $1+1)$; then $9(63=6+3)$, and then back to $7(70=7+0)$; after which the series of reductions repeats.

Thus we now have a repeat series of: 7-5-3-1-8-6-4-2-9-7-5-3-1-8-6-4-2-9-etc.; which, interestingly enough, is the series of odd digits in descending order: (9)-7-5-3-1, followed by a series of even descending digits: 8-6-4-2; in the first repeat, the 9 will of course always be in the 9 th position, because the reduction of the digits of any multiple of 9 is always 9 .

Do you see what I have just done? From the external visible series created by the multiplication table of any number, I was on a quest to discern the fundamental, invisible, or Sung Sang element of these numbers. In this case, the invisible Sung Sang aspect is the reducted form of any multiplication table; and we shall see that with prime number reciprocals, the visible-invisible Hyung Sang - Sung Sang aspects are similarly present, but also mirrored/reversed. The phenomenon of mirrors kept showing itself in my number analysis and will become more apparent. Thus, true to Unification Thought, I believe I have found the Sung Sang and Hyung Sang of whole number multiplication tables.

Before I discuss this a bit further, let us first look at numeric differences.

[^2]
## Numeric Differences

Let's now look at this series of the number 7 as our prime example, which is (9)-7-5-3-1-8-6-4-2-9-etc., from a perspective I have not seen applied in number theory, namely that of numeric differences between these consecutive digits:
to go from 7 to a 5 subtract 2 , or -2
from 5 to 3
from 3 to 1
from 1 to 8
subtract 2 , or -2
subtract 2 , or -2
add 7 , or +7
subtract 2 , or -2
subtract 2 , or -2
subtract 2 , or -2
add 7, or +7
subtract 2 , or -2
Thus, most interestingly, we have a total of seven -2 subtractions, with a total of -14 , and we have a total of two +7 (given in bold) additions, for total of +14 . A perfect balance of plus and minus values. A mirror of sorts. A form of yin and yang? Anyone familiar with Unification Thought will not be surprised here. After all, if Unification Thought is true, it should show itself in all kinds of hitherto unexpected places. So far so good!

Thus, these totals balance, or cancel each other out; just a coincidence? No, we shall see that it is not, after we look at the reducted table for all the first nine numbers, which is the foundation for all numbers greater than a single digit (note that the differences in the 3rd row are only between consecutive reductions):

## The number 1

| Regular: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Reducted: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 |
| Differences: | +1 |  |  |  |  |  |  |  | +1 | +1 |
| Minus: $-8(1 \times 1 \times-8)$ |  |  |  |  |  |  | +1 | +1 | -8 |  |
| Totals: | Plus: $+8(8 \mathrm{x}+1)$ |  |  |  |  |  |  |  |  |  |

The number 2

| Regular: | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Reducted: | 2 | 4 | 6 | 8 | 1 | 3 | 5 | 7 | 9 | 2 |
| Differences: |  | +2 | +2 | +2 | -7 | +2 | +2 | +2 | +2 | -7 |
| Totals: | Plus: +14 (7x+2) |  |  |  | Minus: -14 (2x-7) |  |  |  | Perfect Mirrors! |  |

The number 3

| Regular: | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Reducted: | 3 | 6 | 9 | 3 | 6 | 9 | 3 | 6 | 9 | 3 |
| Differences: | +3 |  |  |  |  |  |  |  |  | +3 |

## The number 4

| Regular: | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: |
| Reducted: | 4 | 8 | 3 | 7 | 2 | 6 | 1 | 5 | 9 | 4 |  |  |  |
| Differences: |  | +4 | -5 | +4 | -5 | +4 | -5 | +4 | +4 | -5 |  |  |  |
| Totals: | Plus: $+20(5 \mathrm{x}+4)$ Minus: $-20(4 \mathrm{x}-5)$ |  |  |  |  |  |  |  | Perfect Mirrors! |  |  |  |  |

The number 5

| Regular: | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Reducted: | 5 | 1 | 6 | 2 | 7 | 3 | 8 | 4 | 9 | 5 |
| Differences: |  | -4 | +5 | -4 | +5 | -4 | +5 | -4 | +5 | -4 |
| Totals: | Plus: $+20(4 \mathrm{x}+5)$ Minus: $-20(5 \mathrm{x}-4)$ |  |  |  |  |  |  |  | Perfect Mirrors! |  |

The number 6

| Regular: | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Reducted: | 6 | 3 | 9 | 6 | 3 | 9 | 6 | 3 | 9 | 6 |
| Differences: |  | -3 | +6 | -3 | -3 | +6 | -3 | -3 | +6 | -3 |
| Totals: | Plus: $+18(3 \mathrm{x}+6)$ Minus: $-18(6 \mathrm{x}-3)$ |  |  |  |  |  |  |  |  | Perfect Mirrors! |

The number 7

| Regular: | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Reducted: | 7 | 5 | 3 | 1 | 8 | 6 | 4 | 2 | 9 | 7 |
| Differences: |  | -2 | -2 | -2 | +7 | -2 | -2 | -2 | +7 | -2 |
| Totals: | Plus $:+14(2 \times+7)$ Minus: $-14(7 \times-2)$ |  |  |  |  |  |  |  |  | Perfect Mirrors! |

## The number 8

| Regular: | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Reducted: | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 9 | 8 |
| Differences: |  | -1 | -1 | -1 | -1 | -1 | -1 | -1 | +8 | -1 |
| Totals: | Plus: $+8(8 \times+1)$ Minus: $-8(1 \times-8)$ |  |  |  |  |  |  |  |  | Perfect Mirrors! |

I have omitted the table of 9 , as reductions of multiples of 9 are always 9 . Let's also look at the results of the reductions in more easy to see table:

| The table of 1: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | Etc. | A1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| The table of 2: | 2 | 4 | 6 | 8 | 1 | 3 | 5 | 7 | 9 | 2 | 4 | 6 | 8 | Etc. | B1 |
| The table of 3: | 3 | 6 | 9 | 3 | 6 | 9 | 3 | 6 | 9 | 3 | 6 | 9 | 3 | Etc. | C1 |
| The table of 4: | 4 | 8 | 3 | 7 | 2 | 6 | 1 | 5 | 9 | 4 | 8 | 3 | 7 | Etc. | D1 |
| The table of 5: | 5 | 1 | 6 | 2 | 7 | 3 | 8 | 4 | 9 | 5 | 1 | 6 | 2 | Etc. | D2 |
| The table of 6: | 6 | 3 | 9 | 6 | 3 | 9 | 6 | 3 | 9 | 6 | 3 | 9 | 6 | Etc. | C 2 |
| The table of 7: | 7 | 5 | 3 | 1 | 8 | 6 | 4 | 2 | 9 | 7 | 5 | 3 | 1 | Etc. | B2 |
| The table of 8: | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 9 | 8 | 7 | 6 | 5 | Etc. | A2 |

The rows of 1 and 8 (note that $1+8=9$ ) are mirrors, as they run in opposite directions (A1 and A2). The same is also true for B1 and B2, C1 and C2 and D1 and D2. Please take a careful look at this.

## Four-Directional Mirrors

Below I have placed two lines, in the form of an "x": one from the left bottom corner to the top right corner, and the other from the top left corner to the bottom right corner, thus effectively dividing our square of reductions into four triangles. When we compare the numbers in each of these four compartments, we will see again Yang-Yin types of symmetrical mirrors:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 4 | 6 | 8 | 1 | 3 | 5 | 7 |
| 3 | 6 | 9 | 3 | 6 | 9 | 3 | 6 |
| 4 | 8 | 3 | 7 | 2 | 6 | 1 | 5 |
| 5 | 1 | 6 | 2 | 7 | 3 | 8 | 4 |
| 6 | 3 | 9 | 6 | 3 | 9 | 6 | 3 |
| 7 | 5 | 3 | 1 | 8 | 6 | 4 | 2 |
| 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |

Thus compare numbers in the top triangle: $\boldsymbol{\nabla}$ with the left triangle: $\boldsymbol{\square}$, then with the right triangle: $\mathbf{4}$, and finally with the bottom triangle: $\boldsymbol{\Delta}$ in order to see the mirror "images" appear. One example is given below, and I invite the reader to discover the others. The left triangle and the top triangle:

| 1 |  |  |  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 4 |  |  |  |  | 4 | 6 | 8 | 1 | 3 | 5 |  |
| 3 | 6 | 9 |  |  |  |  | 9 | 3 | 6 | 9 |  |  |
| 4 | 8 | 3 | 7 |  |  |  |  | 7 | 2 |  |  |  |
| 5 | 1 | 6 | 2 |  |  |  |  |  |  |  |  |  |
| 6 | 3 | 9 |  |  |  |  |  |  |  |  |  |  |
| 7 | 5 |  |  |  |  |  |  |  |  |  |  |  |
| 8 |  |  |  |  |  |  |  |  |  |  |  |  |

If we interpret these two halves as a type of yintyang, then the notion of equality between yin and yang in all its forms and levels, as stated in DP/UT, would automatically be confirmed. If you look carefully along a central vertical and horizontal line (like so: $\dagger$ ), you will see even more mirrors.

Also, when the values are read diagonally (along this line: /), we are discovering palindromes, like 1; 2-2; $3-4-3$; etc., and when read in mirror image (like so: $\backslash$ ), we discover palindromes of 8; 7-7-; 6-5-6; etc. I now invite you to see these and all kinds of other patterns along vertical and diagonal lines, and I have added a checkered pattern to aid you visually:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 4 | 6 | 8 | 1 | 3 | 5 | 7 |
| 3 | 6 | 9 | 3 | 6 | 9 | 3 | 6 |
| 4 | 8 | 3 | 7 | 2 | 6 | 1 | 5 |
| 5 | 1 | 6 | 2 | 7 | 3 | 8 | 4 |
| 6 | 3 | 9 | 6 | 3 | 9 | 6 | 3 |
| 7 | 5 | 3 | 1 | 8 | 6 | 4 | 2 |
| 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |


| We immediately see the appearances of palindromes in the diagonals: |  |  |
| :---: | :---: | :---: |
| From the left top corner, <br> as well as from the right <br> bottom corner, its exact <br> mirror: | And the other diagonal, <br> from the right top corner <br> and from the left bottom <br> corner (its mirror): | Added (sums <br> reducted) (Note: 9 <br> remains central): |
| 1 | 8 | 9 |
| $2-2$ | $7-7$ | $9-9$ |
| $3-4-3$ | $6-5-6$ | $9-9-9$ |
| $4-6-6-4$ | $5-3-3-5$ | $9-9-9-9$ |
| $5-8-9-8-5$ | $3-1-9-1-4$ | $9-9-9-9-9$ |
| $6-1-3-3-1-6$ | $3-8-6-6-8-3$ |  |
| $7-3-6-7-6-3-7$ | $2-6-3-2-3-6-2$ | Etc. |
| $8-5-9-2-2-9-5-8$ | $1-4-9-7-7-9-4-1$ |  |
| $7-3-6-7-6-3-7$ | $2-6-3-2-3-6-2$ |  |
| Etc. | Etc. |  |
| (continues in reverse | (continues in reverse |  |
| order) | order) |  |
| Hence, we have horizontal palindromes, but each group |  |  |
| is also a vertical palindrome |  |  |

This may be stretching my analysis a bit here, but during especially the past decade of his life, Rev. Moon's continually emphasized how from a center (God), we can have six (or double of 3) basic directions, as well as dynamics of relationships, namely: front \& back, left \& right, as well as top (up) \& bottom (down). Palindromes, so it seems to me, may very well be the numeric equivalents of such 3dimensional directional relationships. A thorough look at all the possible palindromes, which is always a structure with mirrored directions and a central point (especially palindromes with an odd number of digits), will further reveal this.

In the table below I have indicated the numeric differences in the table of reductions. I have added labels to each row: A1, A2, etc. Notice the correlations between A1 and A2 (1 and 8), B1 and B2 (2 and 7), C1 and C2 ( 3 and 6), and between D1 and D2 (4 and 5) (and we could have added: between 0 and 9 ). Notice again the centrality of the number 9 . Between these pairs we see perfectly mirrored plus and minus values:

| No. 1: | +1 | +1 | +1 | +1 | +1 | +1 | +1 | A1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| No. 2: | +2 | +2 | +2 | -7 | +2 | +2 | +2 | B1 |
| No. 3: | +3 | +3 | -6 | +3 | +3 | -6 | +3 | C1 |
| No. 4: | +4 | -5 | +4 | -5 | +4 | -5 | +4 | D1 |
| No. 5 | -4 | +5 | -4 | +5 | -4 | +5 | -4 | D2 |
| No. 6: | -3 | +6 | -3 | -3 | +6 | -3 | -3 | C2 |
| No. 7: | -2 | -2 | -2 | +7 | -2 | -2 | -2 | B2 |
| No. 8 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | A2 |

Since $1,10,19,28$, etc. all reduct to 1 , we should also make a table of reductions to 1 , to 2 , to 3 , etc., which is centered again around the number 9 , as each new value is 9 higher than the previous one, as follows:

| Reductions to: | Non-reducted number: |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 10 | 19 | 28 | 37 | 46 | 55 | 64 | 73 | 82 | 91 | 100 | Etc. |
| 2 | 2 | 11 | 20 | 29 | 38 | 47 | 56 | 65 | 74 | 83 | 92 | 101 | Etc. |
| 3 | 3 | 12 | 21 | 30 | 39 | 48 | 57 | 66 | 75 | 84 | 93 | 102 | Etc. |
| 4 | 4 | 13 | 22 | 31 | 40 | 49 | 58 | 67 | 76 | 85 | 94 | 103 | Etc. |
| 5 | 5 | 14 | 23 | 32 | 41 | 50 | 59 | 68 | 77 | 86 | 95 | 104 | Etc. |
| 6 | 6 | 15 | 24 | 33 | 42 | 51 | 60 | 69 | 78 | 87 | 96 | 105 | Etc. |
| 7 | 7 | 16 | 25 | 34 | 43 | 52 | 61 | 70 | 79 | 88 | 97 | 106 | Etc. |
| 8 | 8 | 17 | 26 | 35 | 44 | 53 | 62 | 71 | 80 | 89 | 98 | 107 | Etc. |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 | Etc. |

Observations: many prime numbers appear in this table (bolded) ${ }^{7}$. Also note the palindrome structure in each row (e.g. in the row of 1 , you'll see 19 and 91,28 and 82 , etc.).

## PART TWO: <br> The Dual Aspects of Primes

In Part One I went from the visible whole numbers, like 5, 37, 49, etc. and then dove I into their invisible aspects by reducting them. Thus, from the Hyung-Sang I deducted their Sung-Sang. By this method I was able to show the dual characteristics of whole numbers of the $n / 1$ type. In Part Two we look at numbers, as it were, from their flip-side, what is called their reciprocal side. Allow me to explain what I mean by that and how it translates into Unification Thought terminology.

Let's imagine there are 7 people in a room. Before I can know this, I need to count each person. Thus, when I conclude that 7 is the result, and even write it down on a blackboard, what happens in our conventional way of writing is that we most likely take a most fundamental fact totally for granted, for we just write or say " 7 ". But the truly correct way of writing down our result, as a mathematical formula, should be: $7 / 1$, or 7 divided by 1 , or 7 over 1 . Just to blurt out "seven" is also missing this salient, relationship-centered point.

The 7 people in the room enjoy two types of relationships:

- the group as a whole impacts each individual (relationships of the many to the one); and
- each individual experiences him or herself as part of the group and contributes to the group as a whole (relationship of the one to the many).

In mathematics, the only relationship we see covered, if at all in mathematical dictionaries, is the $1 / n$ type, which in mathematical circles is called the reciprocal (or multiplicative inverse, a term I do not use). I have been very surprised to see how most mathematical dictionaries (if they have an entry on reciprocals at all, as many do not) gloss over the topic and briefly mention it in a short paragraph with an example like $1 / 125$, without writing it out in decimals.

I am taking the mathematical definition of reciprocal a step further. Reciprocity implies a relationship and the relationship between " n " and " 1 ", when staying true to Unification Thought, must be established in

[^3]these two "mirror" formulas: $n / 1$ an $1 / n$. These are "relational mirrors" and form the dual characteristics of a number.

Thus: when the number 7 is given (as $7 / 1$ ) and thus made visible, its reciprocal (as $1 / 7$ ) is not visible and thus remains invisible. The opposite is also true: when I am given the $1 / 7$ value, which is $0.142857 \ldots{ }^{8}$, then the "regular" prime number (as $7 / 1$ ) from which is was derived is made invisible and would need to be figured out.

Number theorists do not consider this dual structure of individual numbers in light of oriental thought, but I think it is quite the correct way of doing so and sets the tone and stage for our further investigations into the dual characteristics of numbers. The reason I am limiting myself to primes (a number can that not be divided any further, like $3,5,7,11,13$, etc.) should be obvious: they are the fundamental building blocks of numbers, as all other whole numbers are composites. Thus, there are two approaches: from the many to the one (as in 123/1) and from the one to the many (as in 1/123).

## Visibility and Invisibility in Prime Reciprocals

The first prime number with an intriguing visible Hyung Sang and invisible Sung Sang aspect in its reciprocal value is the number 7. The number 7 is perfect for demonstrating my findings. Its reciprocal value is simply $0.142857 \ldots$ How do we calculate it? The drawing below illustrates the process:


The outcome of $1 / 7$ is $0.142857 \ldots$...Let's do the same analysis here: 1-4-2-8-5-7-1-etc. provides the following numeric differences: $+3 ;-2 ;+6$, followed by the mirrors of $-3 ;+2$ and -6 . When put into two halves: 1-4-2 added to 8-5-7 gives us 999 (this is always true: the second half of a prime reciprocal added to its first half always adds to a series of 9's).

Take a look at the circled numbers, i.e., the values subtracted in the calculation; when added together in the correct $1 / 10$ th digit offset way, their sum is $0.999999 \ldots$, which, rounded off, brings us back to 1.000....

A yang/yin type pattern exists between the values that were left over after each subtraction, namely: 30-20-60-40-50-10-(30; etc.); the differences between them being: $-10 ;+40 ;-20$; followed by their opposites/mirrors: $+10 ;-40$ and +20 .

The series of these values 7-28-14-56-35-49 - 7 (etc.) can also be seen from the perspective of being either odd ( O ) or even ( E ) and then becomes:

[^4]O-E-E-E-O-O, a mirror when divided into two halves of O-E-E and E-O-O. When we look at the numeric differences between these consecutive values: $+21 ;-14 ;+42$; we see that they are followed by their opposites/mirrors: $-21 ;+14 ;-42$.

Also, if I take the non-circled numbers from each of which is being deducted, namely $1(0), 3(0), 2(0)$, $6(0), 4(0), 5(0)$, and we split this series in half, we have: 1-3-2 and 6-4-5, representing 1 through 6 , and added together as two halves becomes 7-7-7 $(132+645=777)$.

## Dual Characteristics Inside Simple Calculations

Let's return to my handwritten calculation above once more. The solution to $1 / 7=142857 \ldots$. Keep in mind that the number of repeat digits of a prime number reciprocal always follows the formula of $n-1$. Thus, the number 7 has 6 digits in its reciprocal repeat section. The prime number 17 therefore would have 16 digits in one repeat section of its reciprocal $(1 / 17=0.0588235294117647 \ldots$; the starting zero is indeed also counted as one of the digits), etc.

An inherent flaw? We saw that the reciprocal value of $0.142857 \ldots$ seems to have a flaw: why is the 57 not a 56 , as it would then more logically be the double of 28 , which in turn is the double of 14 ? Let's assume for a moment that the value is not 57 , but 56 indeed. And let's also continue to assume that the doubling continues, but that for some mysterious reason we don't see it (yet).

As before, we need to keep one rule in mind: 14,28 , and 56 all have just two digits, while the doubled values that follow after it start with three digits and will grow to many more, so I cannot just write 14-28-56-112-224-etc. as one continuous value (hyphens added); after all, what we get to see is only $0.1427857 \ldots$ I don't visibly see any values over 57. If they are part of it, they must be very hidden.

Hence, I am faced with a seemingly enormous problem, but which is actually easily resolved, if I adhere to a simple rule: I cannot make the illegal move and break the rule of only a two-digit advancement to the right, as 14-28-57 are values that are two digits to the right of the previous value!

This is most important and forces me to create some kind of overlap of digits, and this is best illustrated in a table, whereby each new value is indeed two spaces to the right, and whereby we see that some digits come "underneath" a previous value; hence the overlap, but keep in mind that this analysis is still in a stage of dissection (zeros added for clarity):

| 14 |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 28 |  |  |  |  |  |  |  |  |
|  |  | 56 |  |  |  |  |  |  |  |
|  |  | 01 | 12 |  |  |  |  |  |  |
|  |  |  | 02 | 24 |  |  |  |  |  |
|  |  |  |  | 04 | 48 |  |  |  |  |
|  |  |  |  |  | 08 | 96 |  |  |  |
|  |  |  |  |  |  | 17 | 92 |  |  |
| Totals below: |  |  |  |  |  | 35 | 84 | Etc. |  |
| 14 | 28 | 57 | 14 | 28 | 57 | 14 | $(28)$ | $(57)$ | Etc. |

Thus, it is precisely because of these carefully considered and placed "overlaps" that the value of 142857...indeed appears! What's fascinating here is that apart from the fixed visible reciprocal of $142857 \ldots$ and the eternally growing invisible values of 14-28-56-112-224-etc. underneath it, which is its
most fundamental structure, we also simultaneously witness a different kind of pair system and dual characteristics.

I will call the (largely; we only see the very beginning) invisible multiplication table of 14-28-56-112-224-448-etc. the Sung Sang of the prime number reciprocal: we don't really get to see it; but it's really there, hiding invisibly underneath it, but this Sung Sang is the very raison d'être, the very foundation and basis for the visible manifestation of it, what I would call the HYUNG SANG of the reciprocal. The Sung Sang and HYUNG SANG cannot exist without each other. The invisible is made visible, and its visible form cannot come about without the invisible. There are actually several other methods to construct the reciprocal of 7, and they all follow the same rule of hidden and revealed manifestations.

Thus the simple looking 142857, and thus any prime number reciprocal for that matter, represents a far greater hidden reality, and please pay attention how I carefully word this (and you may have to read this definition more than once to grasp its meaning and implication fully):

The visible numerical value of a prime number reciprocal not only hides the internal hidden invisible structure, but is also its only possible visible manifestation. The invisible aspect is its root, its very foundation. The two aspects belong together and thus form a pair, one is visible, the other is (largely or completely) invisible.

## Subject and Object in Unification Thought

It may be safely assumed that the invisible Sung-Sang of a prime number reciprocal stands in the Subject position, following Unification Thought, as the fundamental multiplication series is primary, and that the Hyung Sang aspect, its visible manifestation, stands in the Object position.

Likewise, it may be safe to assume that the Sung-Sang aspect of whole numbers in their reducted form (e.g. 5 as a form of 14) stand in the Subject position, and that the whole numbers, which are non-reducted, stand in the Object position.

However, it is impossible to designate the + values in the numeric differences as Subject or the - minus differences as Object; after all, the plus or minus result totally depends whether you move from left to right, or from right to left, as both directions are possible, even though we only followed the western convention of left to right. Even the multiplication tables of prime number reciprocals can be read as division tables from right to left as well! This is especially beautifully displayed in the reciprocal of the prime 19 , as $1 / 19=0.052631578947368421 \ldots$, where you see a multiplication table running in the opposite direction from right to left! But it's obviously a division table at the same time, when viewed from the opposite direction. Due to overlap, both divisions and multiplications soon become invisible.

Are there any yang-yin patterns in the Sung Sang aspect of prime number reciprocals, one might ask? Yes, there are, but again, the only way to see them is by reducting the values involved, and here follows thus another basic definition, which we already saw above in Part One: all multiplication tables, when reducted, reveal perfect plus/minus, yang-yin patterns, as well as many palindrome mirrors.

Thus, without the fundamental basis of the invisible Sung Sang, the Yang and Yin could never become manifest. In Unification Thought it is stated that all attributes of dual characteristics are totally ONE within God, but are outwardly manifested in Creation. The same holds true for numbers: the invisible possesses all the inherent attributes that can be discerned as existing outwardly. I can only say that this is a most mysterious thing to watch: I keep asking myself; how can a simple multiplication series give birth
to so many mirrors, palindromes, absolute patterns, Yang-Yin patterns, and many more aspects which I have no space for to cover here? And yet, that's the way it just is. Numbers are what they are.

## Two Types of Reduction or Overlap

The Hyung Sang of a prime number reciprocal is in fact also a form of reduction of its Sung Sang original: after all, we apply the rule of not allowing progressive placement to the right to be more than the number of digits of the starting number, which we saw demonstrated above when we dealt with the reciprocal of 7 . Thus, in a table of columns and rows, we see how digits from consecutive rows are placed in the same column and will be added together, by the process of reduction, as all addition is a form of reduction.

Let me give another example, by introducing the following series: 1-3-9-27-81-243-729-etc. (based on a x3 multiplication, which mathematically includes the formula of: $3^{0}, 3^{1}, 3^{2}, 3^{3}, 3^{4}, 3^{5}$, etc. ), I have to place them properly in a table, like so:

|  | A | B | C | D | E | F | G | H | I | J |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AA | 1 |  |  |  |  |  |  |  |  |  |
| BB |  | 3 |  |  |  |  |  |  |  |  |
| CC |  |  | 9 |  |  |  |  |  |  |  |
| DD |  |  | 2 | 7 |  |  |  |  |  |  |
| EE |  |  |  | 8 | 1 |  |  |  |  |  |
| FF |  |  |  | 2 | 4 | 3 |  |  |  |  |
| GG |  |  |  |  | 7 | 2 | 9 |  |  |  |
| HH |  |  |  |  | 2 | 1 | 8 | 7 |  |  |
| II |  |  |  |  |  | 6 | 5 | 6 | 1 |  |
| JJ |  |  |  |  |  |  |  |  |  |  |
| Totals: |  |  |  |  |  |  |  |  |  |  |
| 1 | 9 | 6 | 8 | 3 |  |  |  |  |  |  |
|  | 1 | 4 | 2 | 8 | 5 | 7 | 1 | $(4)$ | $(2)$ | $(8)$ |

As we know already, $0.142857 \ldots$ is the reciprocal of the prime number 7 . We have THUS found another way to construct the reciprocal of the number 7, and there are more ways to do so! Since the digits of values have to be placed underneath digits in the rows above, which $I$ have called a process of overlapping, I am in fact practicing a process of reduction on a vertical level numbers. In Column E, for example, I am already reducting four digits of rows EE through HH.

Thus, I can take any whole prime number, put it properly into a multiplication table with rows and columns, with the proper decimal offset factors included, and end up with a prime number reciprocal! And the Hyung Sang value of the reciprocal can only become visible via the process result of overlapping and reduction of the Sung Sang values.

Now let's turn to whole numbers, as discussed in Part One. What we just saw above actually mirrors perfectly with what we find with whole numbers. We know that any reduction can represent a basically infinite series of original whole numbers, as e.g. 1, 19, 28, 37, but also 9485763859375937589346, etc. all reduct to 1 . Other numbers reduct to 2 through 9 . Thus, any number larger than 9 , no matter how large, in their reducted form, belong to a value between 1 and 9.

The reduction process of whole numbers, like 12345 can be approached, as we did above, with the very simple process of addition, as $1+2+3+4+5=15$ and then $1+5=6$. This is child's play, but should also be approached in this overlapping-based reduction process.

First of all, I should break 12345 down into 10000 , 2000, 300, 40 and 5. In prime number reciprocal tables I carefully moved one step or more steps to the right with each new value. In this case, as I am working in mirrors I need to move each last value to the left. Remember how in the table of the reciprocal of 7, I placed a zero in front of 112, as 0112 for clarity, and I have to do the same thing, in reverse, for the number 12345. Thus, I first place a zero behind the 5 and turn into a 50 and then place it underneath the 40 to add to 90 . I then add a zero to 90 to become 900 and add it to 300 to become 1200. Next I turn 1200 into 12000 and add it to 2000 to become 14000 , and then turn it into 140000 and add it to 10000 to become my semi-final result of 150000 ; I can then repeat the entire process in just two steps and turn 50000 into 500000 and add it to 100000 to become 600000 , after which I discard the zeroes to become the simple 6 we already established earlier. needless to say, I can also do the entire process in reverse: turn the 10000 to 1000 , add it to 2000 , to get 3000 ; then turn it into 300 ; add it to 300 to make 600 ; turn or reduce it into 70 and add it to 40 to make 100 . The reduce the 100 to 10 and add 5 to make 15 , and $1+5$ $=6$. An approach from the left or right is equally valid and gives the same end result.

And thus we see that there are parallels between the world of whole numbers (primes) and the world of their reciprocals, and that they mirror each other very well. These opposite, or mirror approaches in turn reiterate Unification Thought's emphasis on the two types of dual characteristics.

Thus, we also see once again that by considering the $1 / n$ and $n / 1$ approach as two sides of one reality is the correct way to approach numbers.

## Having Fun With Numbers

I order to solidify and help you do your own research, I am going to provide a few more examples of how yang-yin structures show up when I start "playing around" with prime number reciprocal values. At this point, little commentary will suffice.

|  | Differences between <br> consecutive digits <br> (note the order) |  |  |
| :--- | :--- | :--- | :--- |
| $1 / 11=0.0909090909 \ldots$ | $+9 ;-9$ |  | A1 |
| $2 / 11=0.1818181818 \ldots$ | $+7 ;-7$ |  | B1 |
| $3 / 11=0.2727272727 \ldots$ | $+5 ;-5$ |  | C1 |
| $4 / 11=0.3636363636 \ldots$ | $+3 ;-3$ |  | D1 |
| $5 / 11=0.4545454545 \ldots$ | $+1 ;-1$ | E1 |  |
| $6 / 11=0.5454545454 \ldots$ | $-1 ;+1$ | E1 | E2 |
| $7 / 11=0.6363636363 \ldots$ | $-3 ;+3$ |  | D2 |
| $8 / 11=0.7272727272 \ldots$ | $-5 ;+5$ |  | C2 |
| $9 / 11=0.8181818181 \ldots$ | $-7 ;+7$ |  | B2 |
| $10 / 11=0.9090909090 \ldots$ | $-9 ;+9$ |  | A2 |

Note that the total of each mirror pair, like B1 and B2 is always $11 / 11$ or $0.9999999999 \ldots=1.0000 \ldots$. The vertical arrows show an overall mirror as well. There is a descending order of odd numbers (9-7-5-31), followed by an ascending order, and notice the Yang-Yin and mirror correlation between A1 and A2, B 1 and B2, etc.

My second game is to show you some of the pervasive nature of prime number reciprocal series, as well as the Yang-Yin balances, by showing what happens with a somewhat unusual approach: I reduce the digits of $1 / 7$, which is $0.142857 \ldots$ first to its first digit, then it expand it to two digits, then three, etc., and multiply them with a random number (you can try this approach with a different factor and see what you will get):

| $142857 \ldots$ | Factor <br> chosen: | Result: | Differences from <br> the far right digit to <br> the next below; <br> notice the mirrors |
| :--- | :--- | :--- | :--- |
| 1 | $\times 7=$ | 7 | +1 |
| 14 | $\times 7=$ | 98 | -4 |
| 142 | $\times 7=$ | 994 | +2 |
| 1428 | $\times 7=$ | 9996 | -1 |
| 14285 | $\times 7=$ | 99995 | +4 |
| 142857 | $\times 7=$ | 999999 | +2 |
| 1428571 | $\times 7=$ | 9999997 | -2 |
| 14285714 | $\times 7=$ | 99999998 | +1 |
| Etc. |  | Etc. | Etc. |

Study to see what results you will have if you did things the other way around: focus on the last digit of 142857 and then expand to two digits, etc., and then multiply by 7 ; and thus: first $7 \times 7$, then $57 \times 7$, followed by $857 \times 7$, etc. You will discover some interesting results, and a total of +10 and -10 in the numeric differences between, this time, the first digits of the results. Also, see what happens with these calculations: $1 / 7,2 / 7,3 / 7$, etc. The perfect nature of the prime number 7 will become even more apparent to you!

Many more "games" are possible and I invite my readers to start "playing" with these numbers as you wish.

## Final Observations

So, what can we conclude from this rather brief introduction into the fascinating world of the two types of dual characteristics found in prime numbers?

1. As prime numbers are the fundamental building blocks of all numbers, our study has to start with them.
2. Prime numbers have two aspects that need to be studied as $n / 1$ and also as $1 / n$, that is, from a relational, or reciprocal perspective. Thus prime numbers are like coins with two mirror sides. We study from the many to the one, as well as from the one to the many. I treat them as two sides of one reality. They are also mirrors of each other.
3. Basic math consists of multiplication tables. These tables can be studied from two perspectives: their outward visible values, their Hyung Sang, and via the process of reduction, we discover their inner core or Sung Sang values.
4. In the reducted tables of whole numbers, we discover the first evidence of yang-yin type mirrors and also palindromes.
5. When moving to the $1 / n$ aspect of a prime, or its reciprocal as it is called in mathematics, we discover perfect yang-yin mirrors in the calculation process (values taken out and values left over), as well as in the final result of such calculations.
6. Furthermore, we also discover in the prime number reciprocal a hidden structure, which is the foundation for the visible value. This hidden multiplication series, in light of Unification Thought, I have called the Sung Sang, and the outward visible appearance, and the result of the $1 / n$ calculation, its Hyung Sang.
7. The Sung Sang and Hyung Sang are co-dependent: the Sung Sang cannot express itself without the HYUNG SANG, and the HYUNG SANG cannot exist or be expressed without the Sung Sang. The invisible and the visible are two sides of ONE reality.
8. The Sung Sang is the Subject, while the Hyung Sang is its Object.
9. We also discovered that the process of reduction is present in the Sung Sang structure becoming its visible Hyung Sang expression, in prime number reciprocals, caused by the inevitable overlaps.
10. The additional phenomena of palindromes are an integral part of the yang-yin mirrors we see in plenitude.
11. Even though the written form of a number can only be two-dimensional, it is especially in these palindromes that we seemingly discover multi-directionality.
12. We have discovered the centrality of the number 9 .

Personally, I find only confirmation and no conflict between my findings and Rev. Moon's delineation of the two types of dual characteristics: Sung Sang and Hyung Sang as well as Yang and Yin. In actuality, for me, it was exactly because of these dual types of distinctions in Unification Thought that I not only started to see a consistent pattern, but also found it the only possible and correct interpretative model to make sense out of the literally more than a thousand pages of notes I had produced during my initial enthusiastic, but yet unfocused research.

If indeed Unification Thought is true, then evidence of it in all sorts of academic endeavors will eventually be found. As far as my research into the world of numbers is concerned, which also includes a mathematical model of the Unification $\log { }^{9}$ (which is based on prime numbers), among many other discussions, all I can say is this: So far, so good. Thanks to Rev. Moon, I have been graced by being able to add a new dimension to mathematics.

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[^5]
[^0]:    ${ }^{1}$ Dr. Adrian de Groot, a native of The Netherlands, is a first class alumnus of UTS (1977), and received his doctorate from Columbia Pacific University in musicology. Acting upon a deep inspiration in 2002, he set out to study the world of numbers, especially as how they relate to Unification Thought and presented his findings at the International Symposia of Unification Thought in Bulgaria and Japan. About his most recent book, The Secret World of Numbers, Lulu Press, 2011 (see www.numbersecrets.net), Dr. James Yorke, Distinguished University Professor of Mathematics and Physics at the University of Maryland in College Park, MD wrote: "Here's a primer written by a mathematical artist who by just even simple arithmetic means has discovered a goldmine of beautiful hidden patterns in numbers that so far have gone unrecognized in broad daylight. A fascinating read for anyone into math and numbers."
    ${ }^{2}$ Journal of Unification Studies, Volume VIII, 2007, pp.125-137.

[^1]:    ${ }^{3}$ In the West, often the expression "Yin and Yang" is used in that order, but in all oriental writing, the order is always Yang-Yin, and since this paper deals with Oriental and Unification Thoughts, I have opted to maintain the traditional order.
    ${ }^{4}$ See Adri de Groot, Number Theory in Light of Unification Thought: Preliminary Findings. Tokyo: Unification Thought Institute, 2005; Richard Heath, The Matrix of Creation. St. Dogmaels: Bluestone Press, 2002; Robin Heath, Sun, Moon, \& Earth. New York: Walker \& Company/Wooden Books, 1999; and especially John Martineau, A Little Book of Coincidence. New York: Walker \& Company/Wooden Books, 2001.

[^2]:    ${ }^{5}$ The 'zero" is a digit, invented in India, and represents a clever way to help us deal with the next repeat of 1 through 9 , by adding a zero to the 1 , making it 10 . Thus 10,20 , etc. indicate the 2 nd and 3 rd, etc. repeat of a fundamental series, namely in this case $1-9$, and thus, we need to keep in mind what modern writing conventions are fundamentally all about. One could argue that the decimal system is from $0-9$, while most believe it is from 1-10.
    ${ }^{6}$ Keep in mind that all additions are already a common form of reduction: when, for example, I add 35 to 29 , I carry over the 10 of the first total of 14 , over to the next column to the left(and we ordinarily ignore the 0 ), so that the $30+20$ elements do not total to 50 , but to 60 , as the total is 64. In reverse, substraction is also a form of reduction. A beautiful example of inherent, but not expressed and yet often partially visible, reduction can be seen in e.g. the table of 11. It goes simply first from 11 to 22, then 33, 44, etc., and after 99 we have values like 110 (add the first one to the end, and you have 11 once again), then at some point 143 (which you can easily see is the expanded, exploded form of 44 ), etc. In other words: the inherent 11-based foundation remains quite discernible.

[^3]:    ${ }^{7}$ See my The Secret World of Numbers why I consider numbers like 25, 49, and other squared primes to be also prime. In brief, they are expansions of the prime number system.

[^4]:    ${ }^{8}$ The three dots after the numeric digits indicate that these digits, after the decimal point, keep repeating themselves into infinity. Thus $1 / 7=$ 0.142857142857142857142857 -etc. The other convention is to place a line over the digits.

[^5]:    ${ }^{9}$ See Peter Plichta, God's Secret Formula: Deciphering the Riddle of the Universe and the Prime Number Code. Rockport, MA: Element Books, 1997; and Adri de Groot, Number Theory in Light of Unification Thought: Preliminary Findings. Tokyo: Unification Thought Institute, 2005.

